Karlsruhe Institute of Technology

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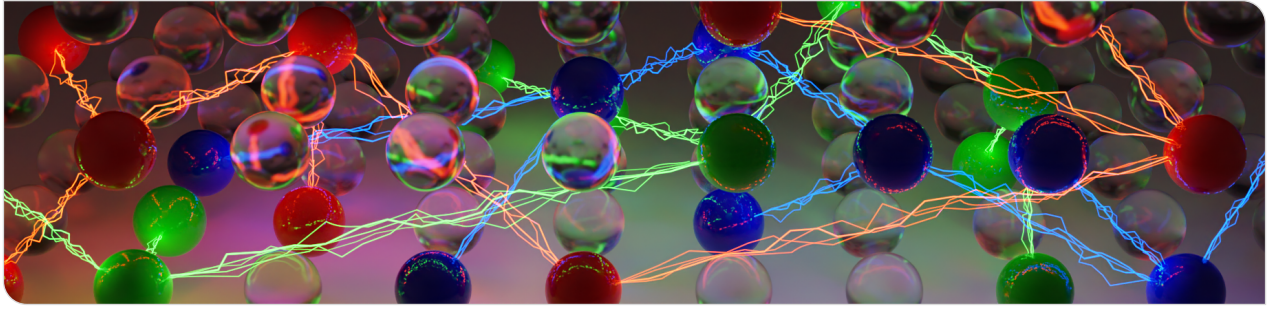
Algorithm Engineering

Decentralized Online Scheduling of Malleable NP-hard Jobs

28th International European Conference on Parallel and Distributed Computing (Euro-Par '22)

Peter Sanders, Dominik Schreiber | August 24, 2022



Motivation: Malleable Job Scheduling

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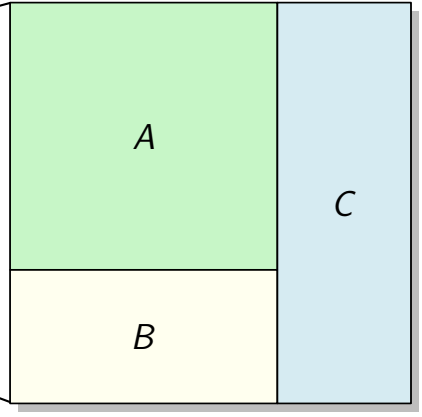

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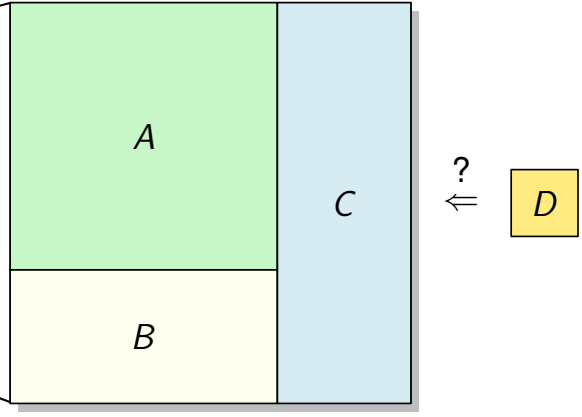

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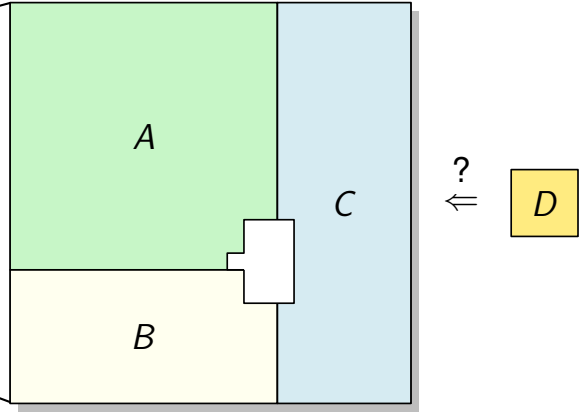

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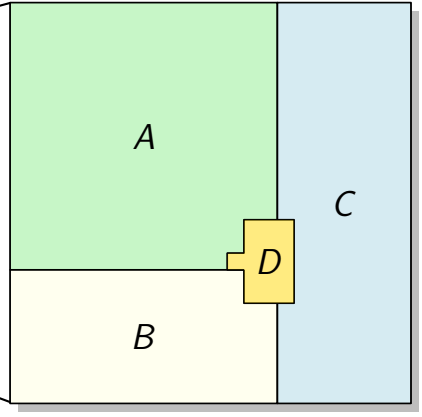

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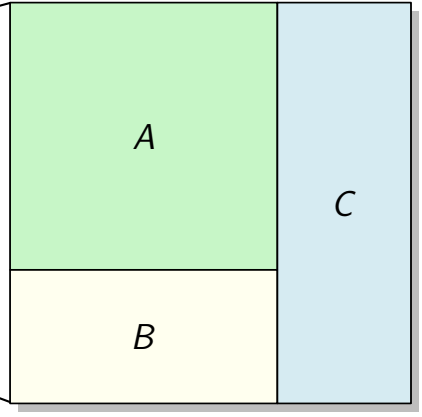

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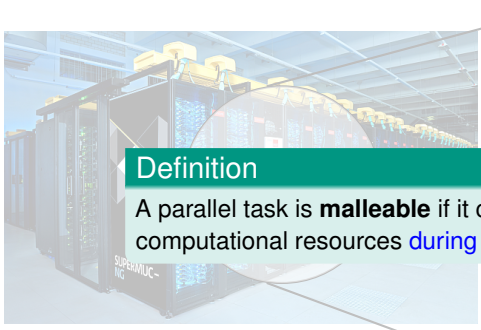

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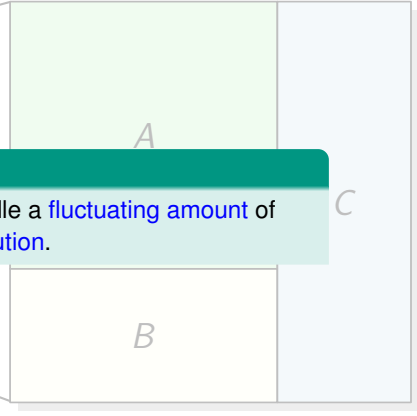


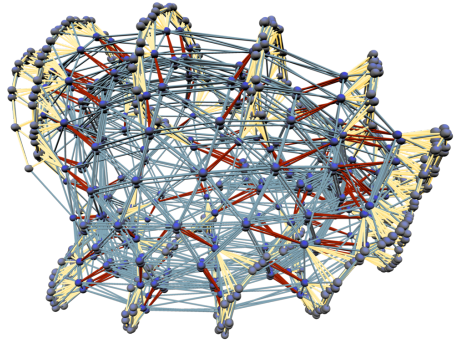
Definition

A parallel task is **malleable** if it can handle a fluctuating amount of computational resources during its execution.

Motivation: SAT Solving

Propositional Satisfiability (SAT)

Input: Propositional formula F (Boolean variables combined with AND, OR, NOT)
Task: Find variable assignment s.t. F evaluates to `true`, or report that no such assignment exists



https://satcompetition.github.io/2022/logo2022-large.png

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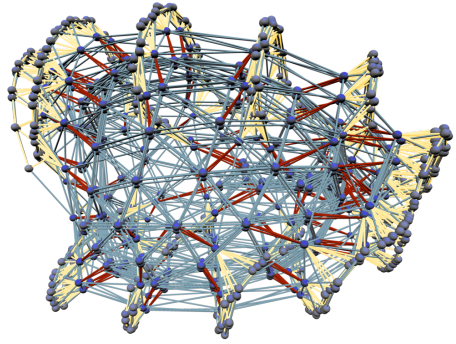
- First problem proven **NP**-complete [Cook '71]
- Crucial buildling block for many applications



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Malleability and SAT Solving: Why?

Particular parallelization

- Portfolio of sequential solvers with diverse search strategies
- All solvers work on the entire problem, exchange knowledge periodically
- Limited scalability (despite significant advancements!)

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- Processing time unknown in advance

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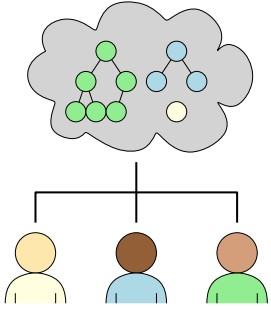
Simple malleability

- Add or remove solvers to/from computation at will
- No global redistribution of data necessary
- Relatively small job descriptions

Vision and Contributions

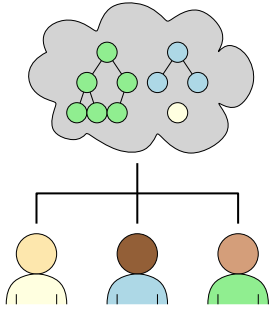


On-Demand Service Platform for NP-hard Problems

Prior work: [Schreiber & Sanders, SAT'21]

- General system architecture
- Preliminary protocols for malleable scheduling
- Focus on award-winning SAT solving engine

Vision and Contributions



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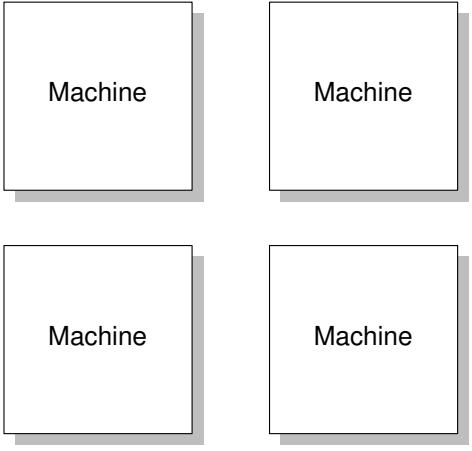
New contributions:

- Fully scalable decentralized scheduling algorithms
- Practical implementation for $\approx 10^4$ cores
- Extensive evaluation of scheduling performance, quality

System Environment



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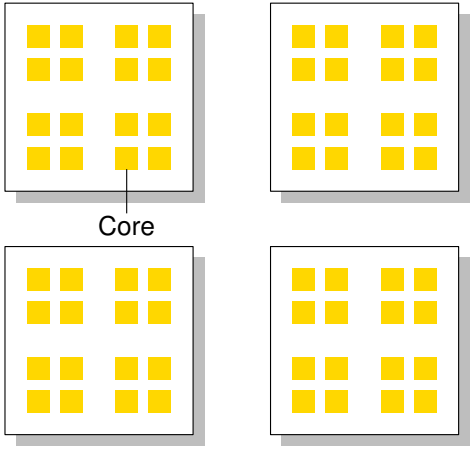

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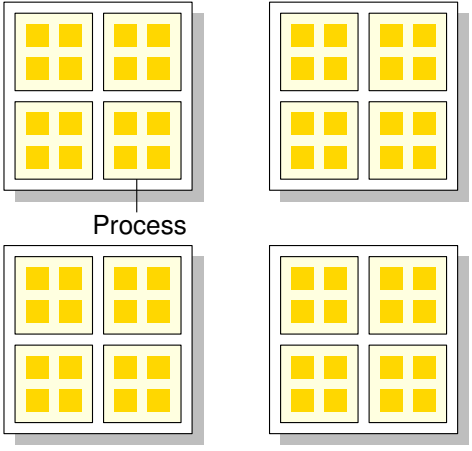
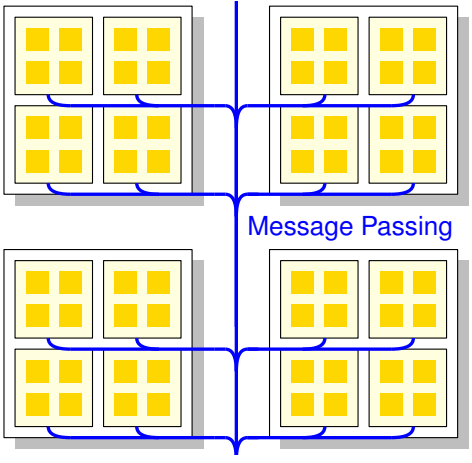


- *m* distributed processes

System Environment



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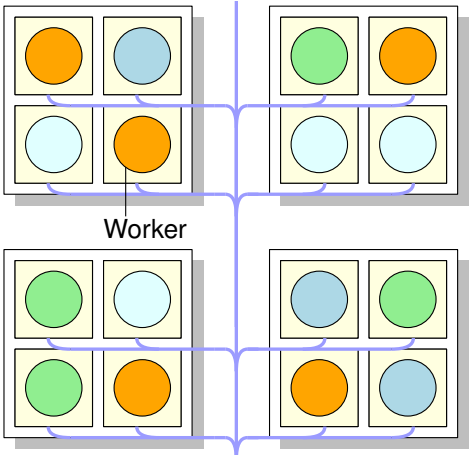

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- Worker: Context of certain job on certain process

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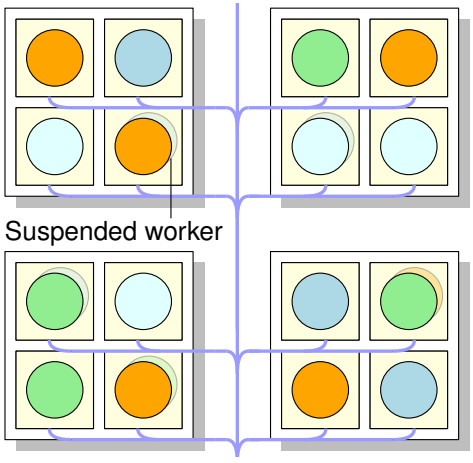

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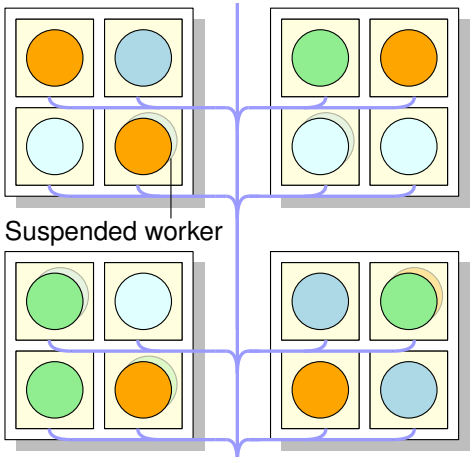

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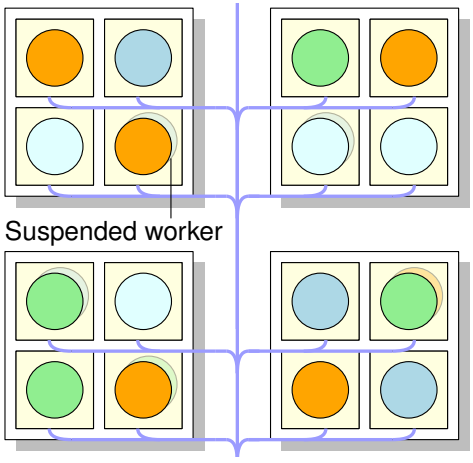

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- Active jobs J, $n := |J|$
- Properties of each job j:
  - Priority $p_j \in \mathbb{R}^+$
  - Demand of resources $d_j \in \mathbb{N}^+$
  - Wallclock or CPU budget $b_j \in \mathbb{R}^+$ (optional)

Assigning Fair Volumes to Jobs

Volume Assignment Problem

Map each job $j \in J$ to a fair number of workers, the volume $v_j \in \mathbb{N}^+$, s.t.

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- Solve unused resources $\xi(\alpha) := m - \sum_{j \in J} v_j(\alpha)$ for $\xi(\alpha) = 0$

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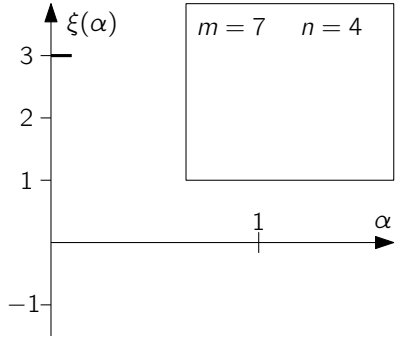

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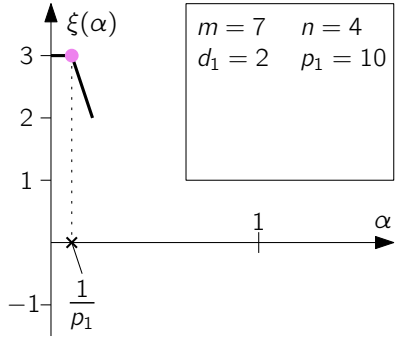

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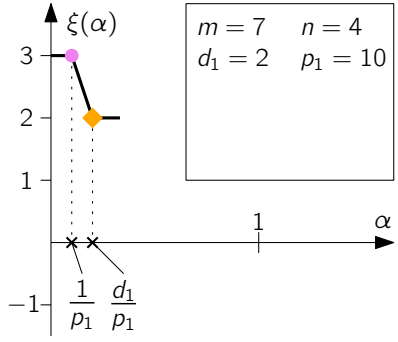

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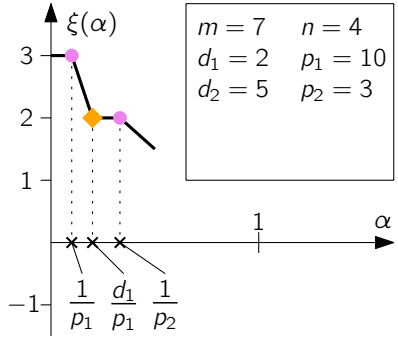

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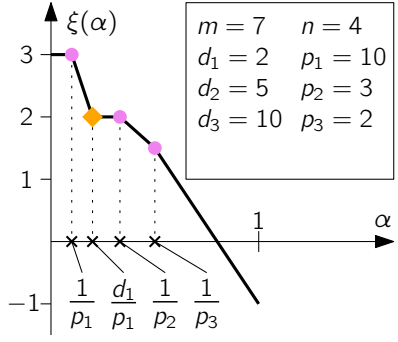

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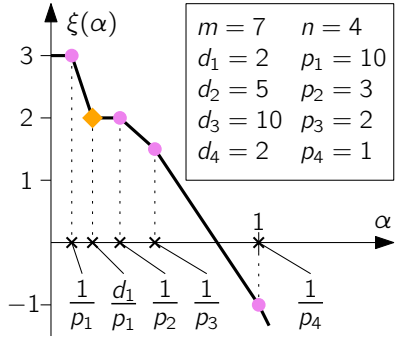

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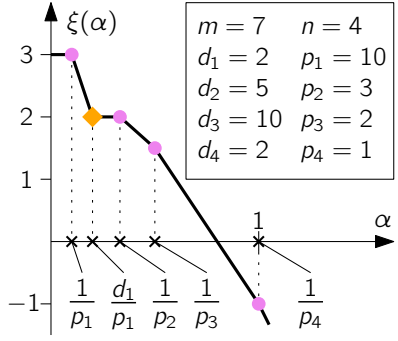

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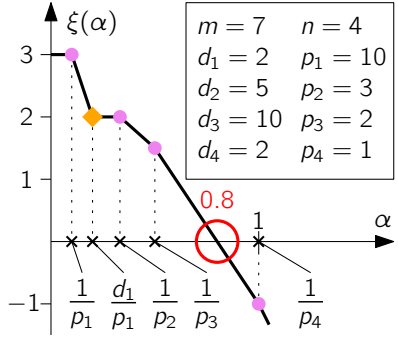

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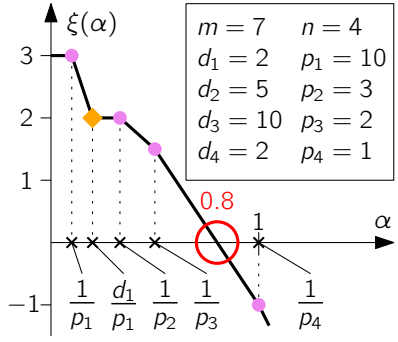

$v_1(\hat{\alpha}) = d_1 = 2$
$v_2(\hat{\alpha}) = 0.8 \cdot 3 = 2.4$
$v_3(\hat{\alpha}) = 0.8 \cdot 2 = 1.6$
$v_4(\hat{\alpha}) = 1$

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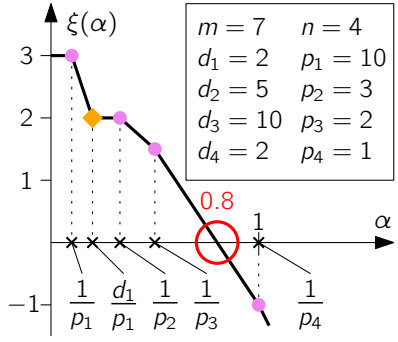

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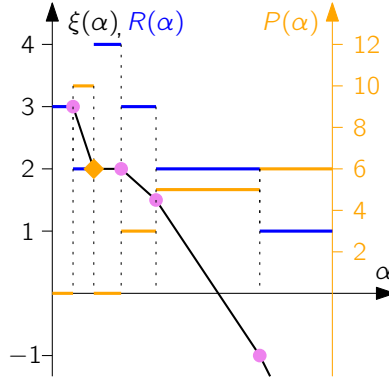
*__Parallel evaluation of ξ:__

- Compute ξ based on auxilliary terms R and P



Volume Assignment Algorithm

- Job volumes $v_j = v_j(\alpha) = \max(1, \min(d_j, \alpha p_j))$, $\alpha \geq 0$
- Excess resources $\xi(\alpha) = m - \sum_{j \in J} v_j(\alpha)$
- Evaluate ξ in parallel* at all $2n$ values where ξ' changes
- Interpolate value $\hat{\alpha}$ where $\xi(\hat{\alpha}) = 0$
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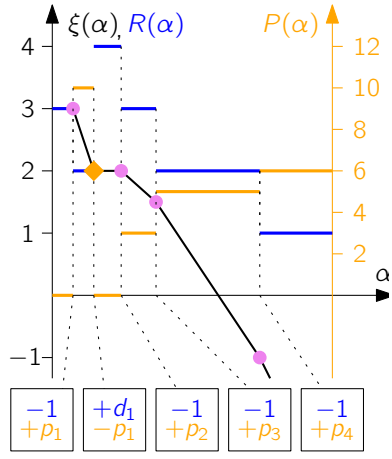

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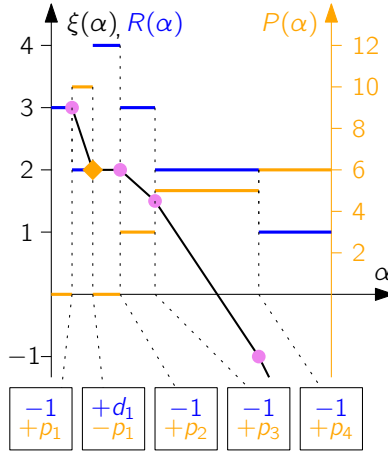

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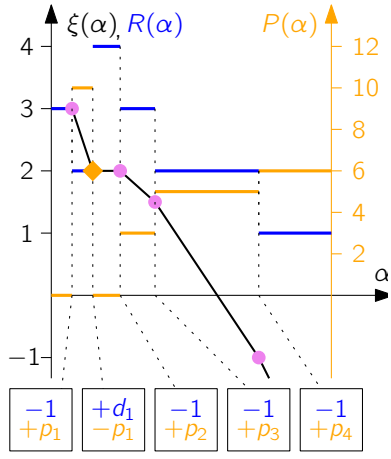
All-reductions, prefix sums, sorting $\mathcal{O}(m)$ elements:
Possible in $\mathcal{O}(\log m)$ time [Ajtai, Komlós, Szemerédi '83]



From Volumes to Scheduling

Each job j receives v_j processes. **Which processes?**

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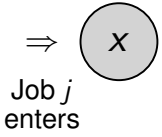

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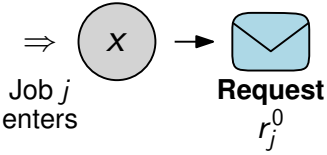

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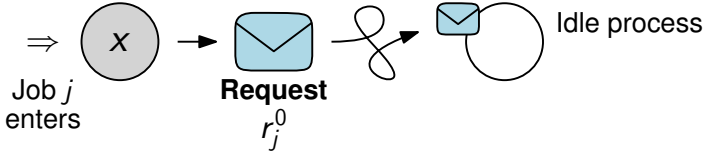

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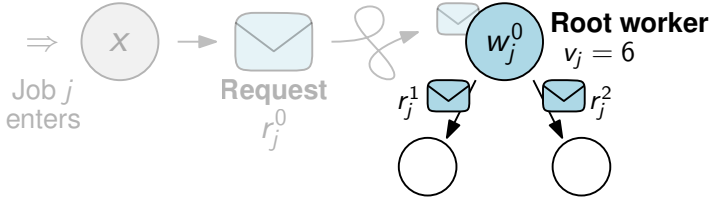

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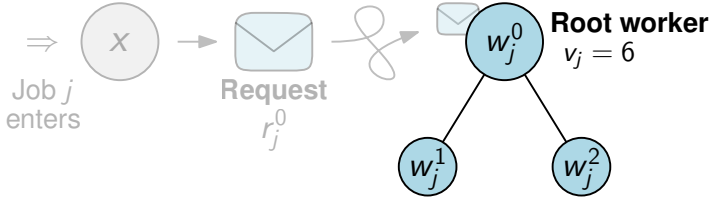

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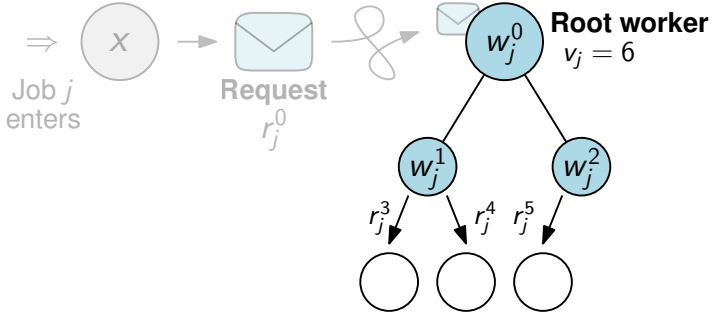

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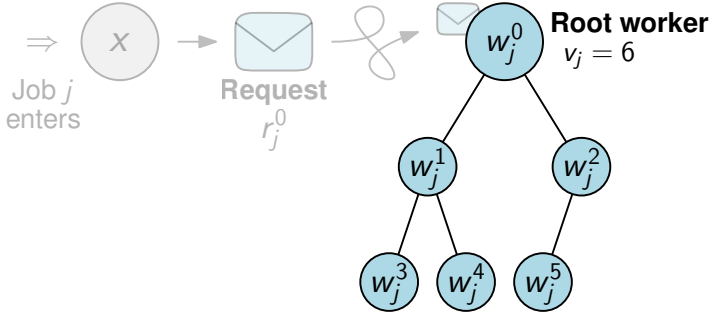

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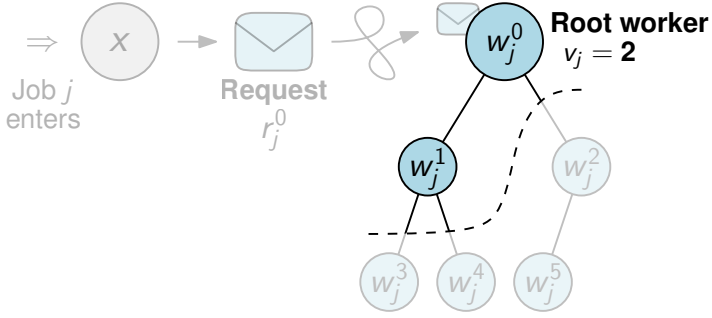

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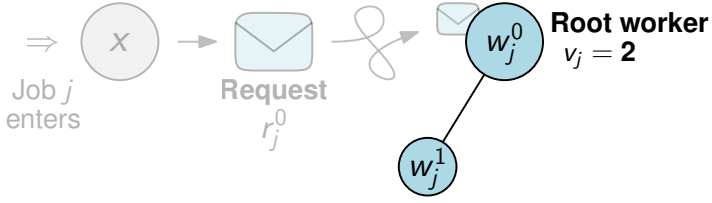

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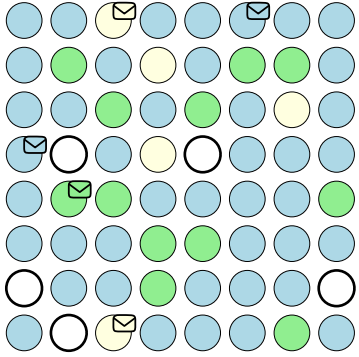
Matching Requests and Idle Processes

Random walks [Schreiber & Sanders, SAT'21]



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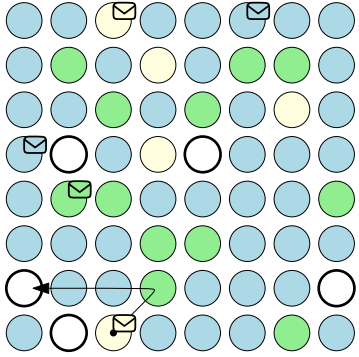

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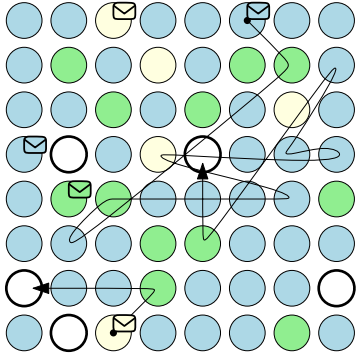

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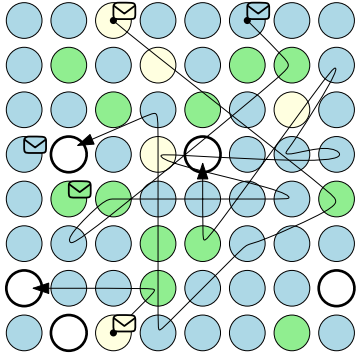

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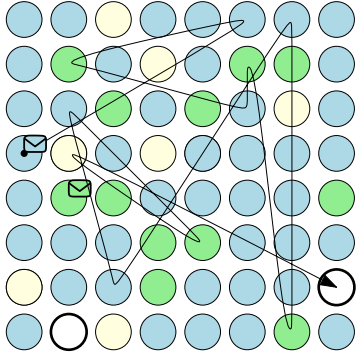

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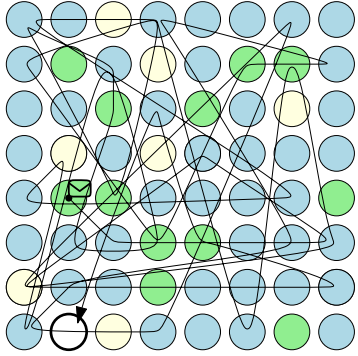

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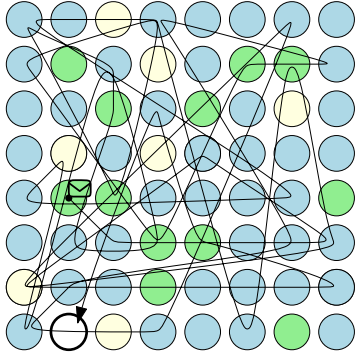

⇒ Deliberately leave some processes idle
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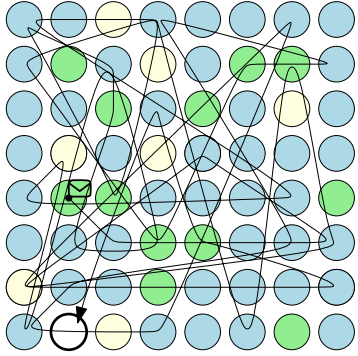

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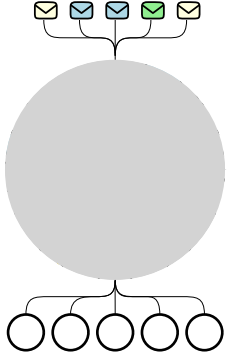

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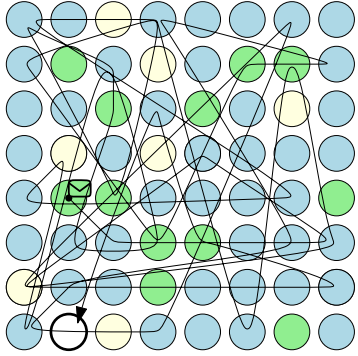

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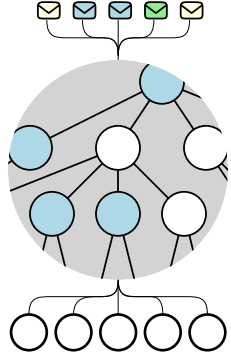

Implemented: Route requests along tree

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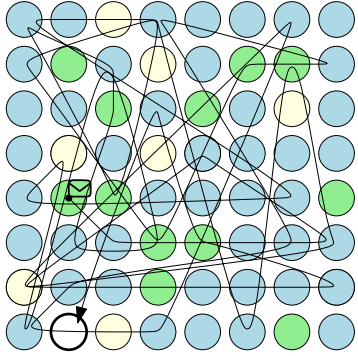

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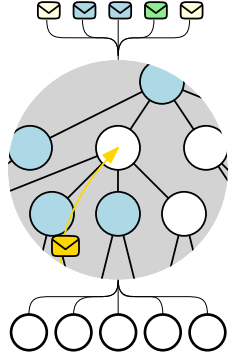

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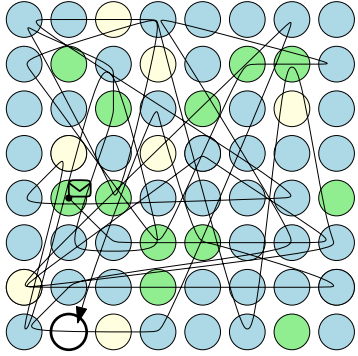

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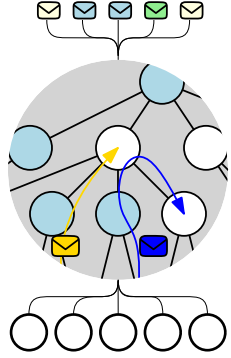

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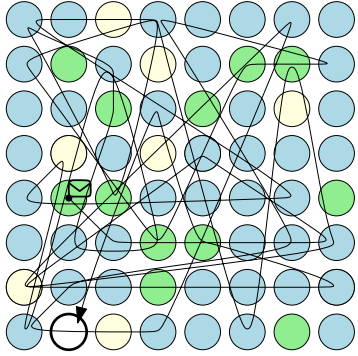

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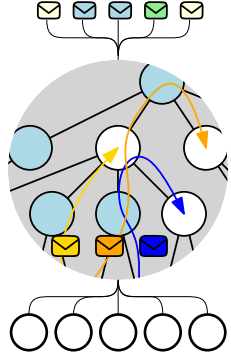

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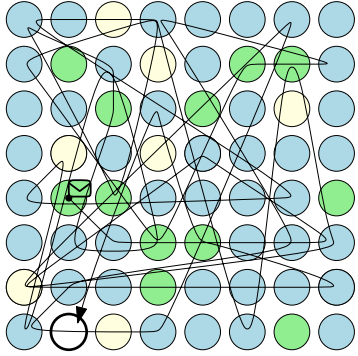

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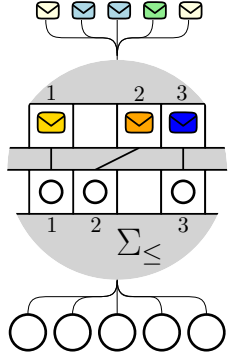
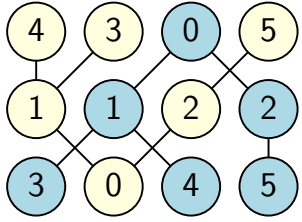


Implemented: Route requests along tree
Theory: $\mathcal{O}(\log m)$ span via prefix sums

Reusing Suspended Workers

Naïve scheduling



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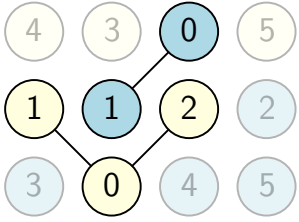

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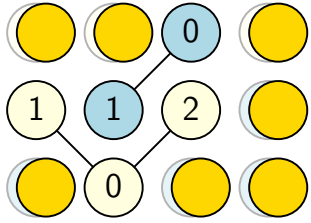

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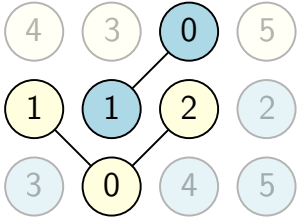

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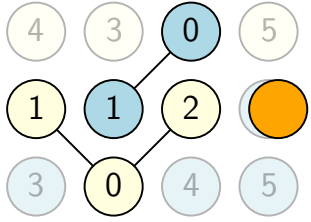

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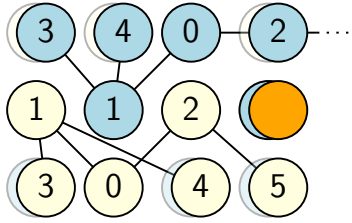

- Idle procs. can be seized by other jobs
- Jobs can re-grow differently after shrinking

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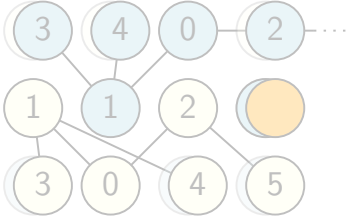
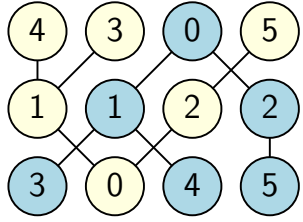


Improvements



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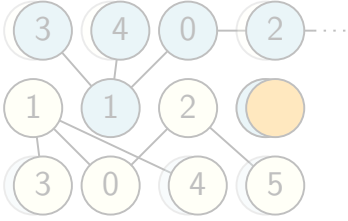

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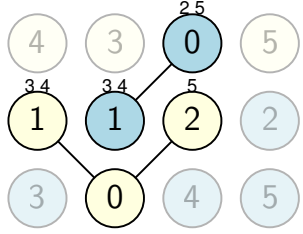

- Workers transitively remember past children

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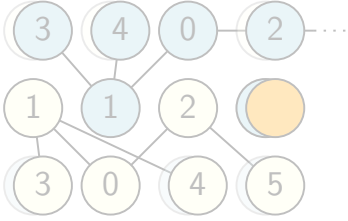

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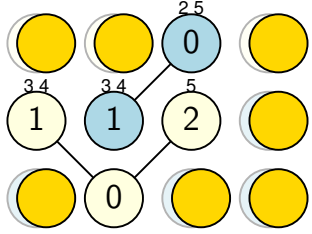

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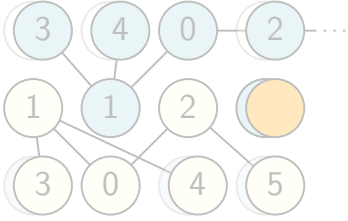

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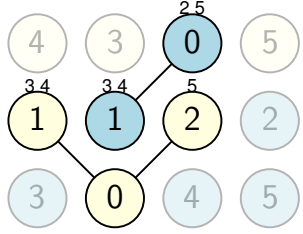

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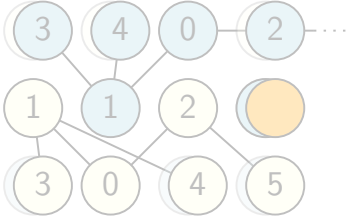

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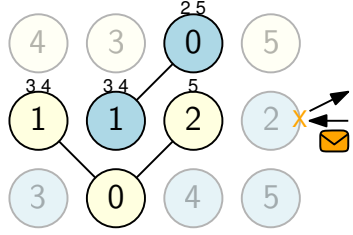

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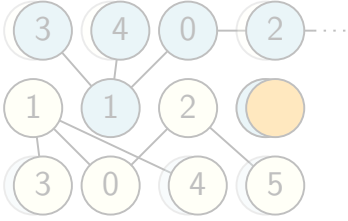
Naïve scheduling



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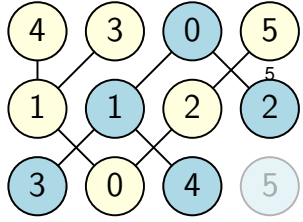

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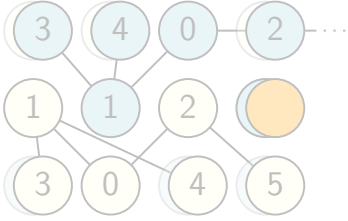

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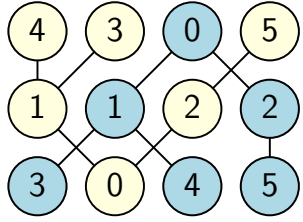

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Now: **Performance and quality of our scheduling**

Evaluation: Setup

Environment

SuperMUC-NG (#26 @ TOP500 '22)
$\leq$ 128 nodes $\times$ 48 cores @ 2.7 GHz
SuSE Linux Enterprise Server

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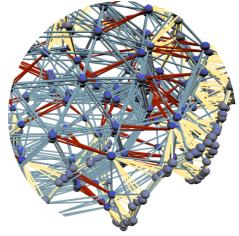
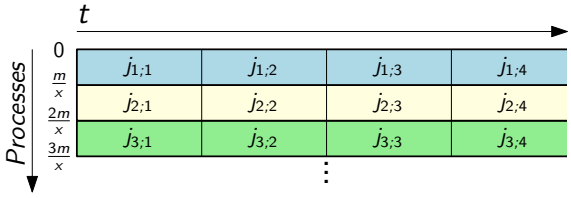


400 instances from
Int. SAT Competition 2020

Efficiency for Uniform Jobs

Setup: 1536 processes $\times$ four cores
Scenario: x jobs in parallel with fixed CPU limit



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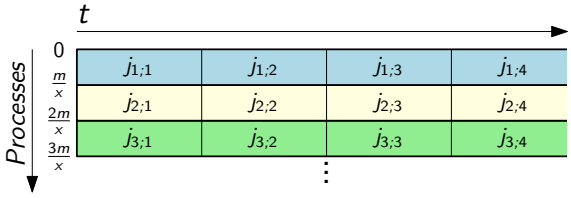
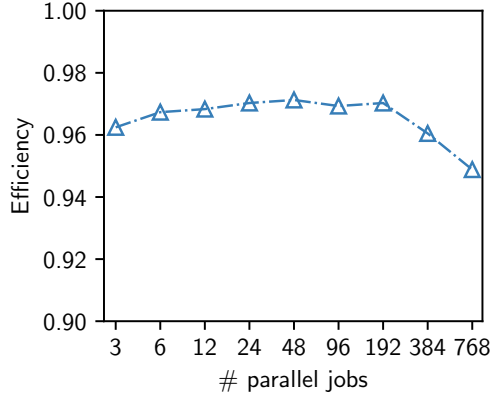


Efficiency: >96% for reasonable loads



Impact of Job Priorities

Setup: 384 processes $\times$ four cores
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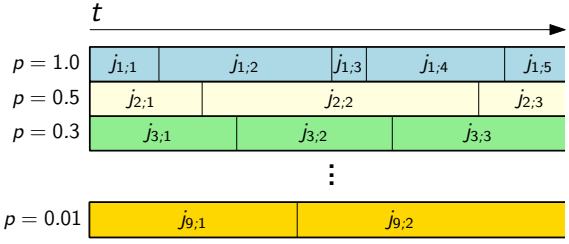

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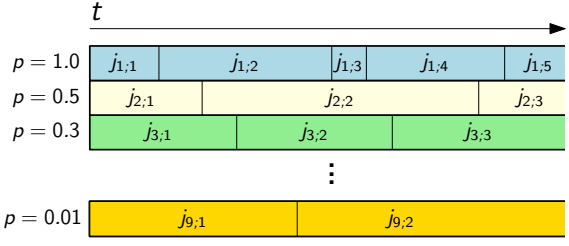

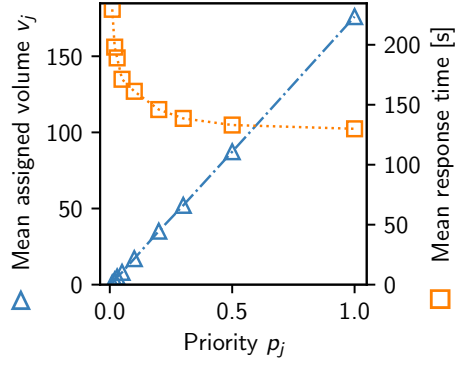

Realistic Arrival Rates: Overview

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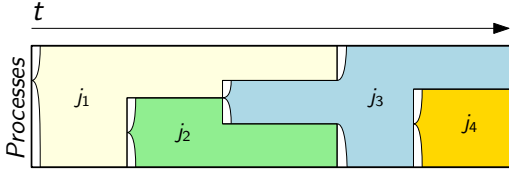

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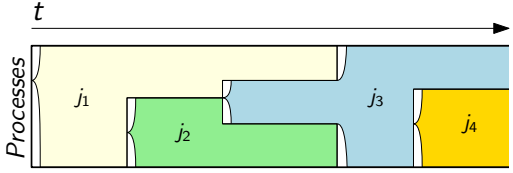

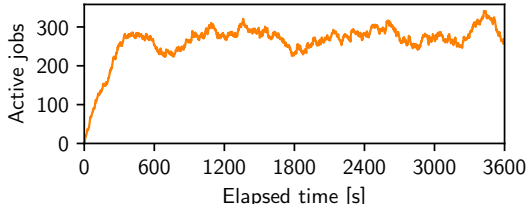

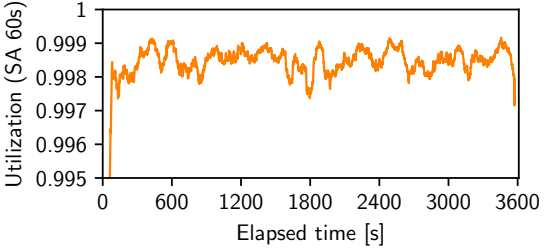

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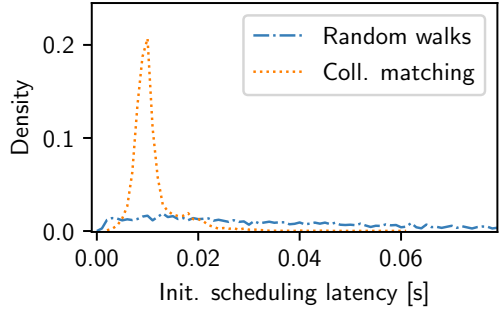

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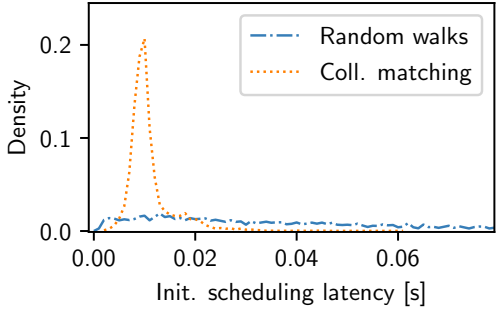

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Conclusion

Recap

- Decentralized online scheduling of malleable jobs with priorities, demands, unknown processing time
- Theory: Scalable algorithms with logarithmic span
- Experiments (6144 c.): Scheduling delays in range of milliseconds, near-optimal utilization

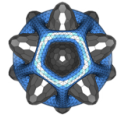
Future work

- Integration of further applications (k-Means clustering, hierarchical planning)
- Fault tolerance, heterogeneous systems
- Better handling of fractional resources

Many thanks to all reviewers!

Certified Euro-Par Artifact

Published at Journal of Open Source Software



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Ready for cloud deployment

Software archived

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/domschrei /mallob

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Collective operations with $\mathcal{O}(\log m)$ *span* / *depth*:

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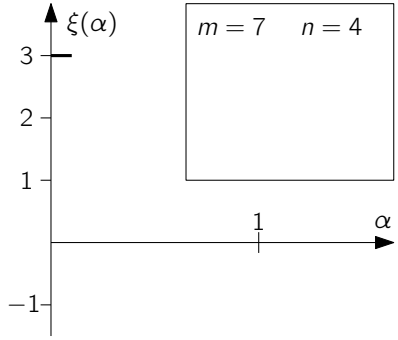

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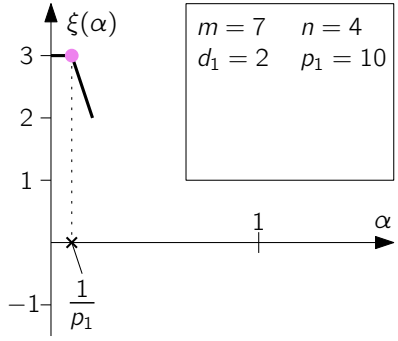

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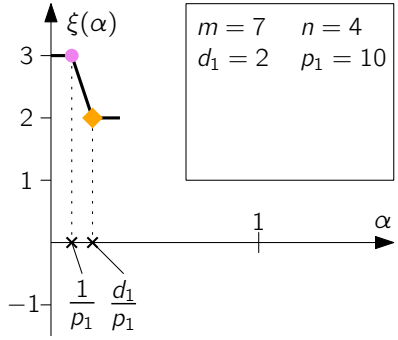

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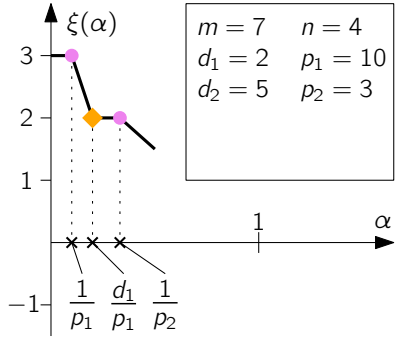

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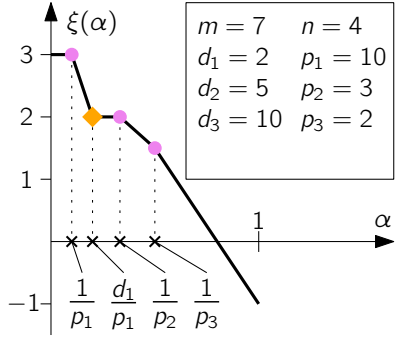

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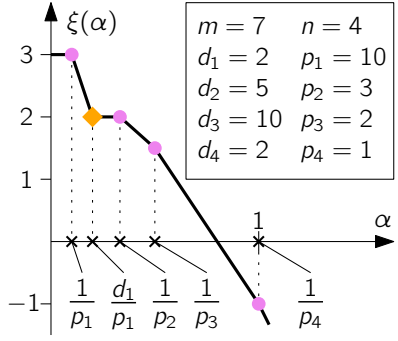

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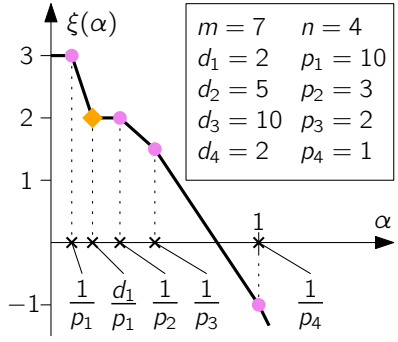

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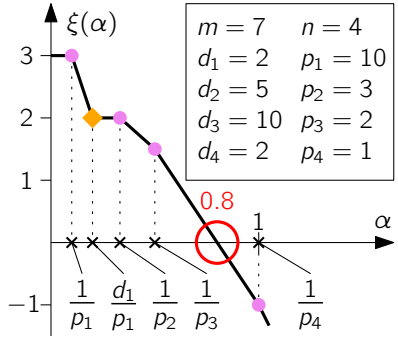

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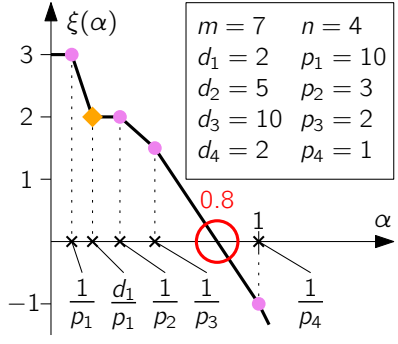

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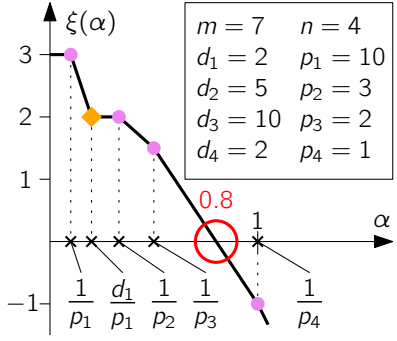

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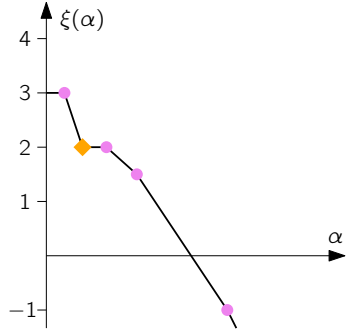

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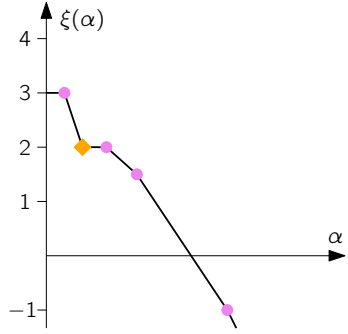

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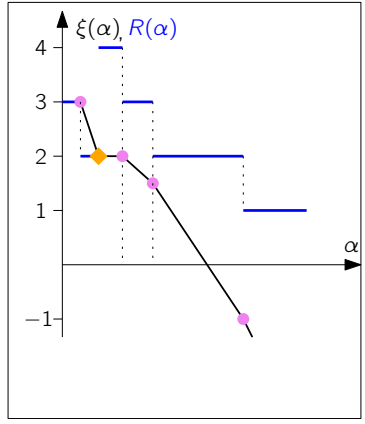

Volume Assignment (2/2)

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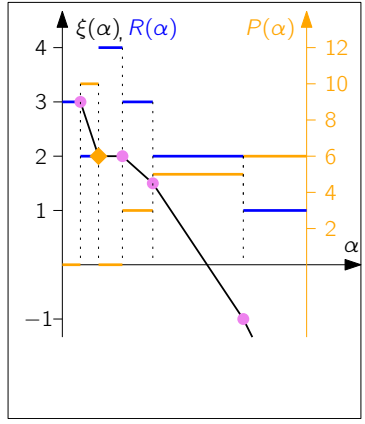

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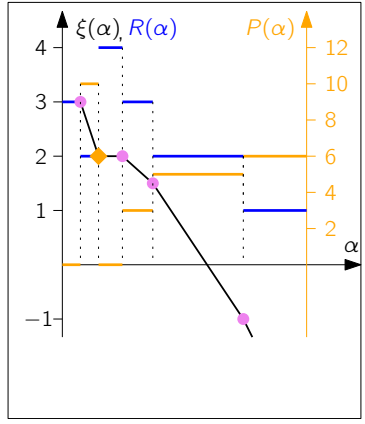

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- Express $2n$ points as **events** manipulating R, P
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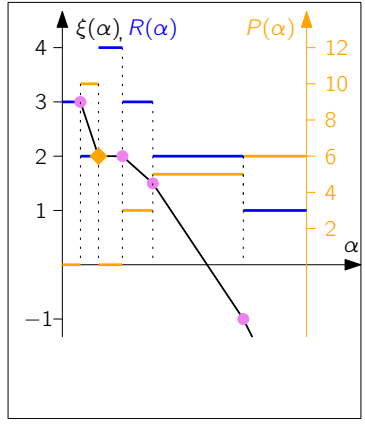
- Compute **prefix sum** $(R_{\le e}, P_{\le e})$ over sorted events e



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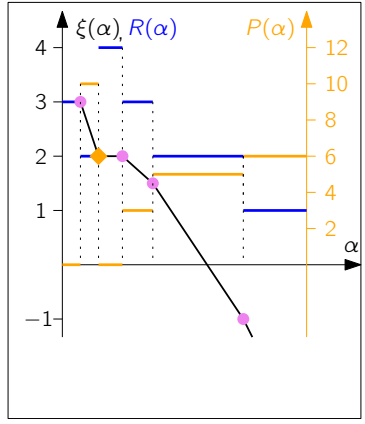
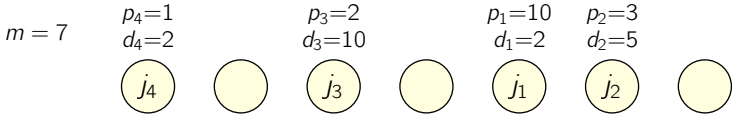
- **Evaluate** $\xi(e) = m - (n + R_{\le e}) - e\,P_{\le e}$



Volume Assignment: Algorithm



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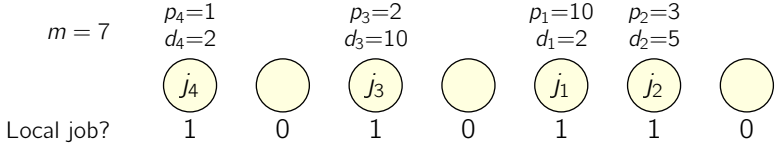

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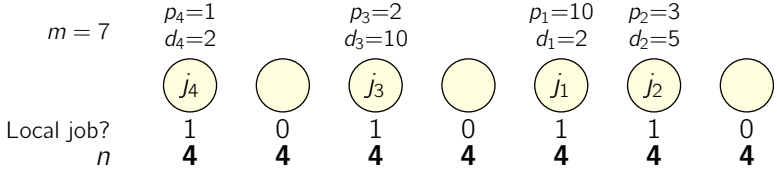

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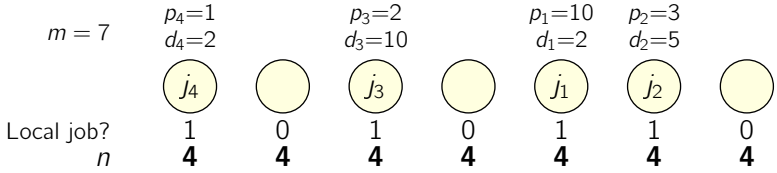

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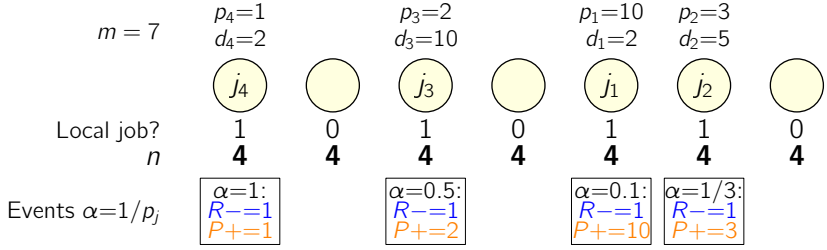

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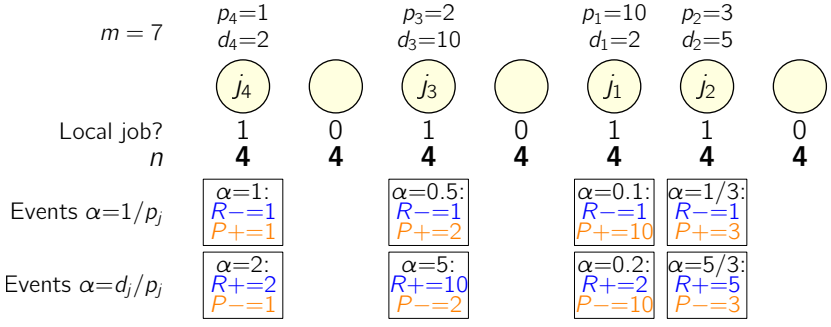

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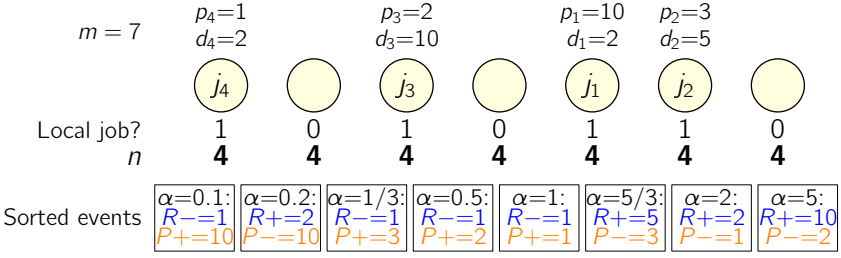

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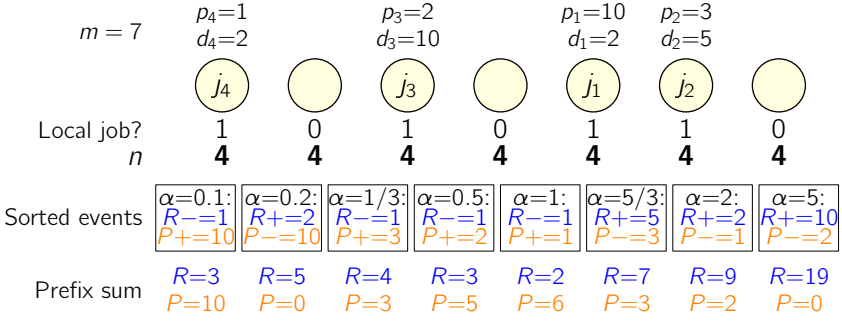

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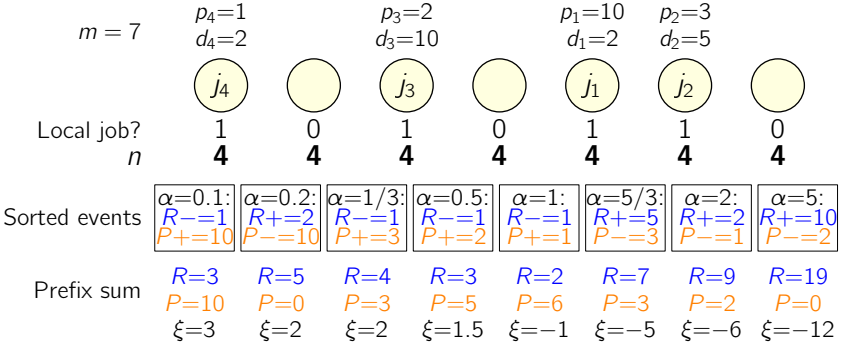

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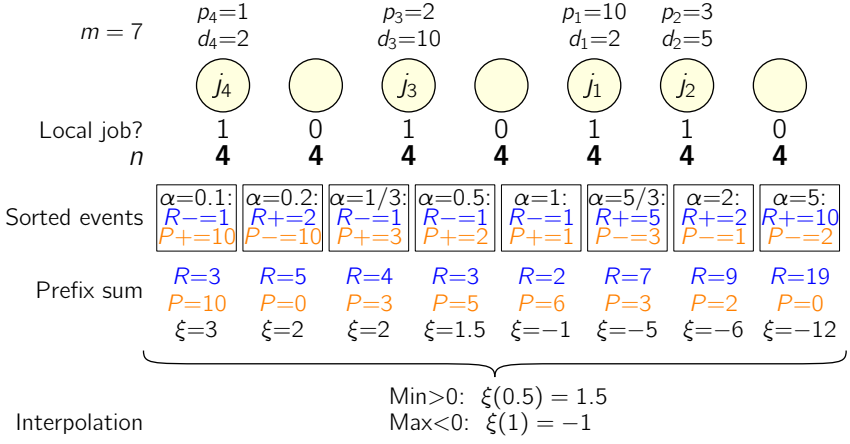

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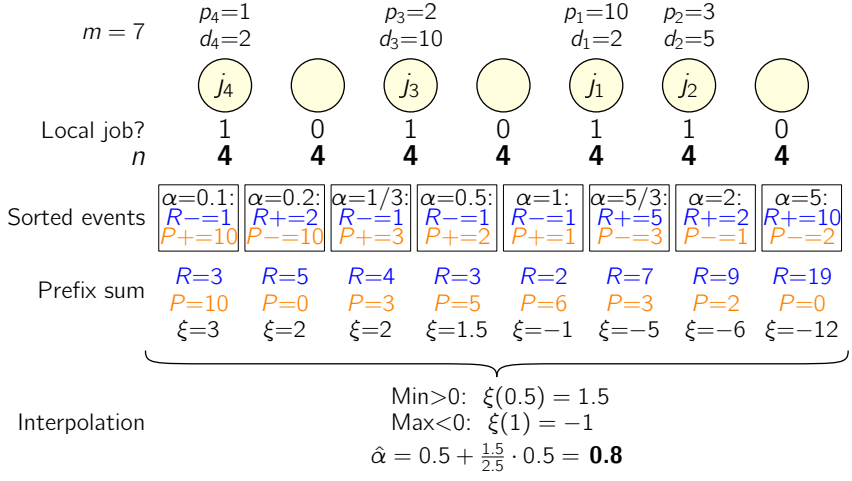

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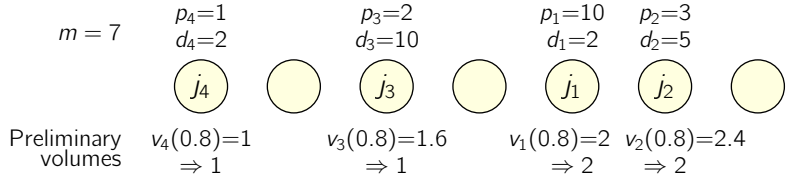

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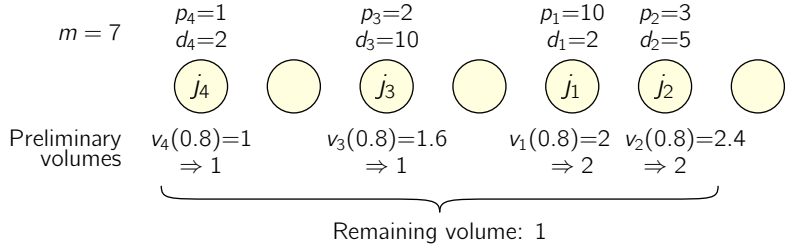

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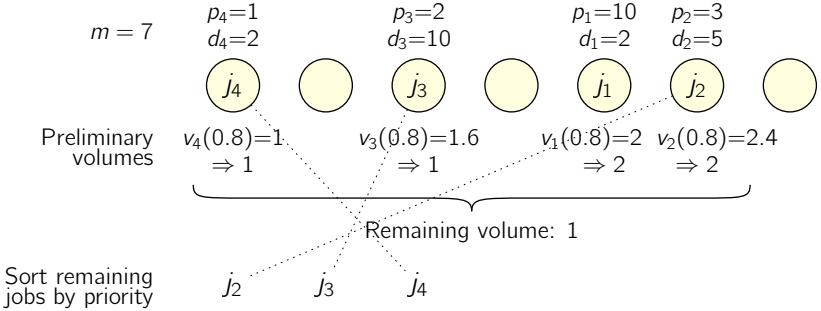

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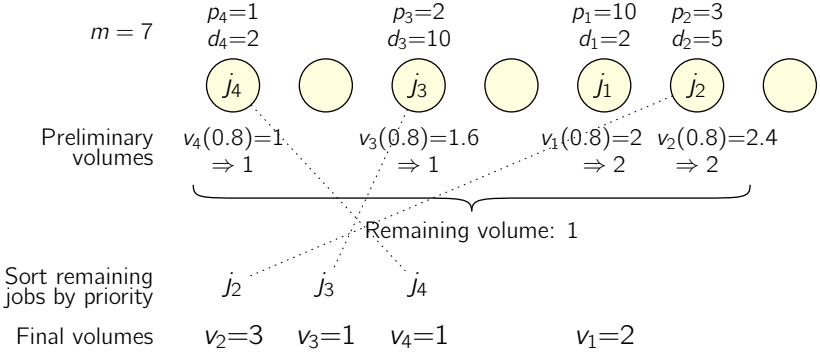

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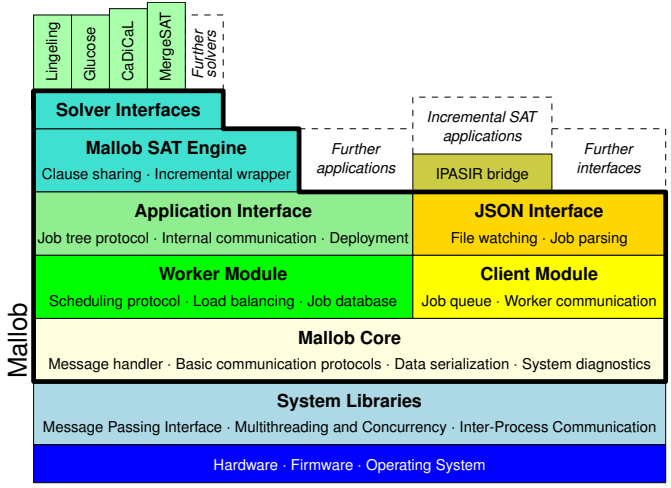
Mallob: Technology Stack



Realistic Arrival Rates: Worker Reuse

Worker reuse stategy	Workers created / Workers required median	max.	total
None	1.43	33.0	2.14
Basic [Schreiber & Sanders, SAT'21]	1.40	31.5	2.07
Ours	**1.25**	**24.5**	**1.80**

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$\Rightarrow$ Most workers ($\approx 90\%$) are initialized only once